



UK Maths Trust

JUNIOR MATHEMATICAL CHALLENGE

Solutions 2024

© 2024 UK Mathematics Trust

supported by

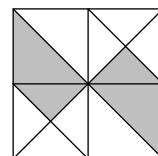


For reasons of space, these solutions are necessarily brief.

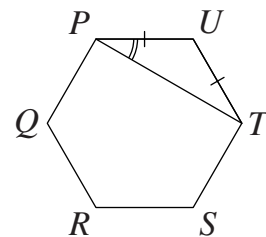
There are more in-depth, extended solutions available on the UKMT website,
which include some exercises for further investigation:

www.ukmt.org.uk

1. **C** Reading from left to right, the values of the five expressions are 4, 4, 0, 1 and 4.
So there are three different values.
2. **B** Of the options given, a cup, a dustpan and an egg would have capacities far smaller than 10 litres, while an aeroplane would have a capacity far greater than 10 litres. However, a bucket could well have a capacity of 10 litres.
3. **B** As 36 is the square of 6, Gill's age will next be a square when it is the square of 7, namely 49. That will be in thirteen years' time - in 2037.
4. **A** The cordial makes up one-fifth, that is 20%, of the drink.
5. **E** Multiplication and division take precedence over addition and subtraction.
Therefore $1 + 2 - 3 \times 4 \div 5 = 1 + 2 - (3 \times 4 \div 5) = 3 - 12 \div 5 = 3 - 2.4 = 0.6$.
6. **E** The lowest common multiple of 3 and 4 is 12 so any number which is 1 more than a multiple of 12 or 2 more than a multiple of 12 has the same remainder when divided by 3 as it does when divided by 4. Of the options given, only 25 satisfies that requirement.
7. **D** Note that the two small shaded triangles have a combined area equal to the area of one of the larger shaded triangles in the diagram. These larger shaded triangles each has an area which is half of a quarter, that is an eighth, of the area of the large square. So the total shaded area is three-eighths of the area of the large square.
8. **C** Let Skye have x pens. Then Ishaa has $2x$ pens, as does Ana. So there are $5x$ pens in total and Skye has one-fifth of these.



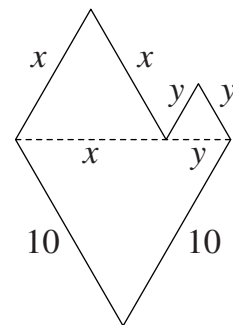
9. A The sum of the interior angles of a polygon with n sides is $(n-2) \times 180^\circ$. So the sum of the interior angles of $PQRSTU$ is $(6-2) \times 180^\circ = 720^\circ$. Therefore, as the hexagon is regular, $\angle PUT = 720^\circ \div 6 = 120^\circ$. The lengths of the sides of a regular polygon are equal, so $PU = UT$ and triangle PUT is isosceles. Hence $\angle UPT = \angle UTP = (180 - 120)^\circ \div 2 = 30^\circ$.



10. E If all those in Fred's field were humans then there would be 25 heads and 50 legs. If one human is replaced by a horse, the number of heads is unchanged but the number of legs increases by 2. Therefore, as there is a total of 60 legs, the number of horses in the field is $(60 - 50) \div 2 = 5$. So there are 20 humans and the required difference is $20 - 5 = 15$.

11. B Let the side-lengths, in cm, of the two smaller equilateral triangles be x and y , as shown.

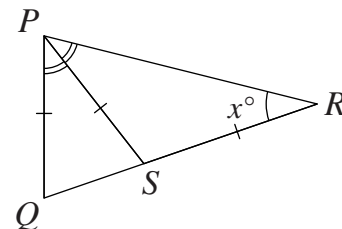
Then, looking at the dotted line in the diagram, we see that $x + y = 10$.



So the perimeter, in cm, of the hexagon is $2 \times 10 + 2x + 2y = 20 + 2(x + y) = 20 + 20 = 40$.

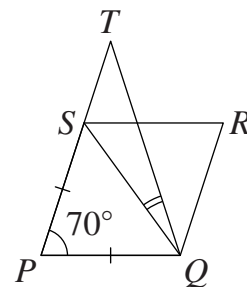
12. E Note that the product of the two positive integers, both having units digit y , also has units digit y . So y is 0, 1, 5 or 6. However, $y \neq 0$ as $30 \times 400 \neq 10770$. To test $y = 1$ and $y = 5$, we first look at the calculations which affect the tens digit of the product, namely 7. As $31 \times 11 = 341$ and $35 \times 55 = 1925$, we deduce that $y \neq 1$ and $y \neq 5$. It then remains for us to check that $y = 6$ by confirming that 36×466 does indeed equal 16 776.

13. C Triangle PSR is isosceles as $PS = RS$. So $\angle SPR = \angle SRP = x^\circ$. Therefore $\angle QPS$ is also x° . From the exterior angle theorem, $\angle PSQ = \angle PRS + \angle SPR = x^\circ + x^\circ = 2x^\circ$. Triangle PQS is also isosceles as $PQ = PS$. Hence $\angle PQS = \angle PSQ = 2x^\circ$. Therefore the sum of the interior angles of triangle PQS is $(2x + 2x + x)^\circ = 5x^\circ$. Hence $5x = 180$. So $x = 36$.



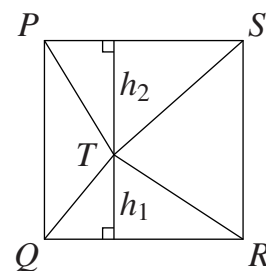
14. E Clearly the divisor (the number immediately after the division sign) in the calculation is not 1 as the result would then be a two-digit number. If the divisor is 2, then the dividend (the number to be divided) could be 18, 38 or 58. However, in each of these cases the final result would be a two-digit number, so the divisor is not 2. If the divisor is 3, then the only calculations which produce a single-digit final result are $12 \div 3 + 5 = 9 \neq 8$; $15 \div 3 + 2 = 7 \neq 8$ and $18 \div 3 + 2 = 8 \neq 5$, so all are incorrect. The divisor cannot be 5 as no two digits from 1, 2, 3, 8 can form a multiple of 5. If the divisor is 8, then the only possibility for the dividend is 32 and $32 \div 8 + 1 = 5$ is indeed correct.

15. **D** As $PQRS$ is a rhombus, PQ is parallel to SR .
 So $\angle SPQ$ is $(180 - 110)^\circ = 70^\circ$. Also, $SP = PQ$ so triangle SPQ is isosceles and $\angle PQS = \angle PSQ = (180 - 70)^\circ \div 2 = 55^\circ$.
 Now $PT = QT$, so triangle TPQ is isosceles and $\angle PQT = \angle QPT = 70^\circ$. Therefore $\angle SQT = \angle PQT - \angle PQS = (70 - 55)^\circ = 15^\circ$.



16. **B** Note that there are 1000 mm in 1 metre. So the number of frogs which would be needed to make a line 1 metre long is $1000 \div 7.7 \approx 1000 \div 8 = 125 \approx 130$.

17. **C** First note that, as the area of square $PQRS$ is 100 cm^2 , its side-length is 10 cm.
 Let the perpendicular heights, in cm, of triangles QRT and PTS be h_1 and h_2 respectively, as shown. Then $h_1 + h_2 = 10$.
 The sum of the areas, in cm^2 , of triangles QRT and PTS is $\frac{1}{2} \times 10 \times h_1 + \frac{1}{2} \times 10 \times h_2 = 5(h_1 + h_2) = 5 \times 10 = 50$.
 Therefore, as the area of triangle QRT is 24 cm^2 , the area of triangle PTS is $(50 - 24) \text{ cm}^2 = 26 \text{ cm}^2$.



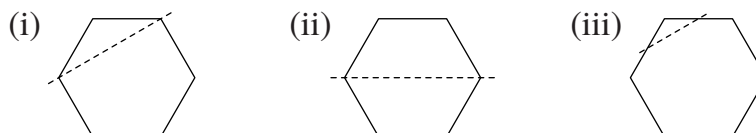
18. **D** Let the amount of porridge in each bowl be p .
 Then, when Goldilocks has eaten $\frac{3}{7}$ of the total amount of porridge, she has eaten $\frac{3}{7} \times 3p = \frac{9p}{7} = p + \frac{2p}{7}$. So Goldilocks will have eaten $\frac{2}{7}$ of the porridge in the second bowl when she has eaten $\frac{3}{7}$ of the total amount of porridge.

19. **C** We first consider the veracity of the statements, assuming that P is a Clown. Then it is true that Q always lies, so Q is a Joker. Q's statement is therefore false, which means that R is a Clown and this fact is consistent with R's statement. However, this means that S's statement is false, so S is a Joker.

Now assume that P is a Joker. Then the statement, "Q always lies" is false, which means that Q is a Clown. The statement that R always lies is therefore true, which means that R is a Joker and this fact is consistent with R's statement. So, in this case, S is telling the truth. Therefore S is a Clown.

Although the above means that we cannot deduce whether any specific one of the four is a Clown or a Joker, we can deduce that exactly two of them are Clowns and the other two are Jokers.

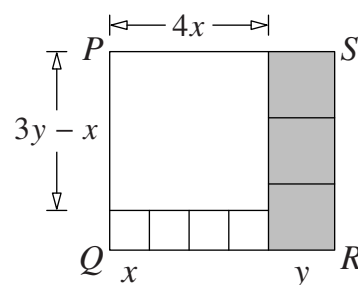
20. **E** When a regular hexagon is cut into two pieces with a single straight cut, for either of the two resulting polygons the number of sides can be at most one more than the number of sides in the original hexagon. So it is possible to obtain a heptagon but not an octagon. Diagram (i) below shows that a triangle and a pentagon may be obtained, diagram (ii) shows two quadrilaterals and diagram (iii) shows how a heptagon may be obtained.



- 21. B** Let the side-length of the small unshaded squares be x and that of the shaded squares be y , as shown. Then the side-length of the large unshaded square may be expressed as $4x$ or as $3y - x$. So $4x = 3y - x$. Therefore $x = \frac{3y}{5}$.

The length of QR is $4x + y = 4 \times \frac{3y}{5} + y = \frac{12y}{5} + y = \frac{17y}{5}$.

Since $PQRS$ and the shaded rectangle have the same height, namely SR , the fraction of the area of rectangle which is shaded is $\frac{y}{4x + y} = \frac{y}{\frac{17y}{5}} = y \times \frac{5}{17y} = \frac{5}{17}$.

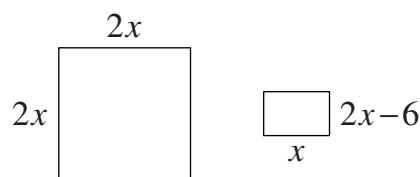


- 22. D** Let the weights of one furry ferret, one fit ferret and one friendly ferret be u, i and r respectively. Then $40u = 50i$, that is $4u = 5i$. Also, $45i = 54r$, that is $5i = 6r$. Hence $4u = 6r$.

So the number of friendly ferrets which weigh the same as fifty furry ferrets is $50 \times \frac{6}{4}$, that is 75.

- 23. D** Let the side-length of the square, in cm, be $2x$. Then the rectangle has length x cm and width $(2x - 6)$ cm.

The area of the square is 6 times the area of the rectangle, so $(2x)^2 = 6 \times x(2x - 6)$, that is $4x^2 = 6x(2x - 6)$.



As $x \neq 0$, this equation simplifies to $4x = 12x - 36$, that is $8x = 36$.

Now the perimeter, in cm, of the square is $4 \times 2x = 8x$. Hence the required perimeter is 36 cm.

- 24. B** Note that United drew two of their matches and lost the third. They scored three goals and conceded six goals, so the match they lost was by a margin of three goals. Rovers won all three of their matches and in doing so did not concede any goals. Therefore the match which United lost was against Rovers and the score was Rovers 3 United 0.

(It is left to the reader to determine the scores in the other five matches.)

- 25. A** Let the number of right-handed pupils be r and the number of left-handed students be l .

As one-fifth of the students have blue eyes, the number of students having blue eyes is $\frac{r + l}{5}$.

Therefore $\frac{l}{10} + \frac{r}{4} = \frac{r + l}{5}$. Multiplying both sides by 20 gives $2l + 5r = 4l + 4r$, that is $r = 2l$.

Hence the total number of students is $3l$, one-third of whom are left-handed.